
A Deep Discriminant Fractional-order Canonical Correlation Analysis For Information Fusion

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Outline

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- The Proposed Method
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- Summary

Background & Motivation

- Information fusion has taken center stage in machine learning and intelligent multimedia research and applications.
- The class based on canonical correlation analysis (CCA) [1] has drawn wide attention.
- The family of CCA inspired methods relevant to this research:
 - kernel CCA (KCCA) [2] and deep CCA (DCCA) [3].
 - deep and discriminative CCA (DDCCA) [4]
 - multi-view fractional deep CCA (MFDCCA) [5]
- State-of-the-art (Discriminant Correlation Analysis (DCA)): Making use of between-class and within-class correlation matrices to extract powerful discriminant information.
- The challenge (in real applications),
 - even though the sample-based matrices are unbiased estimates,
 - the corresponding eigenvalues are biased estimates.

Background & Motivation-cont'

- A potential solution – the fractional-order (FO) algorithm [7].
- Ongoing research shows that integrating Statistics Guided Optimization (SGO) with neural network (NN) architecture (SGO-NN) exhibits model agnostic properties and is ideal for interpretable machine learning [39].
- A deep discriminant fractional-order canonical correlation analysis (DDFCCA) method is proposed by integrating the architecture of NN with FO CCA, a SGO algorithm.
- Functions of FO – correct eigenvalues in the correlation matrices, and then construct FO discriminant correlation matrices.
- Hence, DDFCCA generates high quality information representation and an interpretable model via
 - Effectively extracting the discriminant information according to FO correlation
 - Revealing the intrinsic nonlinear relation via the NN architecture from multiple data/information sources.
 - SGO-NN strategy

DDFCCA (1)

□ Discriminant Correlation Analysis (DCA)

Suppose $x' = [x_1, \dots, x_N] \in R^{m \times N}$ and $y' = [y_1, \dots, y_N] \in R^{p \times N}$ are the two random data sets, where N is the number of samples and m & p are the number of dimensions of x' & y' .

The mean vector values of x' and y' are:

$$x_M = \frac{1}{N} \sum_{i=1}^N x_i, y_M = \frac{1}{N} \sum_{i=1}^N y_i. \quad (1)$$

Then, the two zero-mean variables sets x and y are expressed as

$$\begin{aligned} x &= [x_1 - x_M, \dots, x_N - x_M], \\ y &= [y_1 - y_M, \dots, y_N - y_M]. \end{aligned} \quad (2)$$

DDFCCA (2)

The within-class correlation matrix is $C_{w_{xy}} = xDy^T$ and between-class correlation matrix is $C_{b_{xy}} = -\delta xDy^T$ [8], where δ is a constant and

$$D = \left[\begin{array}{ccc} \left(\begin{array}{ccc} H_{n_1 \times n_1} & \dots & 0 \\ \vdots & H_{n_d \times n_d} & \vdots \\ 0 & \dots & H_{n_c \times n_c} \end{array} \right) \end{array} \right] \in R^{N \times N}, \quad (3)$$

The objective of DCA algorithm – find the two projected matrices W_x and W_y in equation (4)

$$\arg \max W_x^T \tilde{C}_{xy} W_y. \quad (4)$$

where $\tilde{C}_{xy} = C_{w_{xy}} - C_{b_{xy}}$.

Mathematically, Lagrange multiplier and eigenvalue decomposition (GEV) algorithm are utilized to find the solution to (4).

DDFCCA (3)

In order to discover the high level semantic relation across different inputs, a cascade layers-based network is applied to X and Y respectively, resulting in

$$f(X) = g(W_d^X h_{d-1} + b_d^X), f(Y) = g(W_l^Y h_{l-1} + b_l^Y). \quad (5)$$

Then, converting parameters W_i^X, b_i^X and W_j^Y, b_j^Y ($1 \leq i \leq d, 1 \leq j \leq k$) into the vector form leads to

$$\begin{aligned} \theta_X &= [W_1^X, W_2^X, \dots, W_d^X, b_1^X, b_2^X, \dots, b_d^X], \\ \theta_Y &= [W_1^Y, W_2^Y, \dots, W_l^Y, b_1^Y, b_2^Y, \dots, b_l^Y]. \end{aligned} \quad (6)$$

Combining equations (5) and (6) leads to

$$f(X) = f(X, \theta_X), f(Y) = f(Y, \theta_Y). \quad (7)$$

DDFCCA (4)

□ The Proposed Method

A fractional-order operation is performed on the discriminant correlation matrix (\widetilde{C}_{xy}) to extract the discriminant information across multiple data/information sources. Equation (4) is rewritten as follows:

$$\arg \max_{W_x, W_y} W_x^T (\widetilde{C}_{xy})^\alpha W_y, \quad (8)$$

where α is the fractional-order ($\alpha = 0.1, 0.2, \dots, 1$).

Substituting $f(X)$ and $f(Y)$ into (8) leads to

$$\arg \max_{W_{f(X)}, W_{f(Y)}} W_{f(X)}^T (\widetilde{C}_{f(X)f(Y)})^\alpha W_{f(Y)}. \quad (9)$$

To solve the optimization problem in (9), the orthogonality constraints is imposed, leading to the following relation

DDFCCA (5)

$$\arg \max_{W_{f(X)}, W_{f(Y)}} W_{f(X)}^T (C_{f(X)f(Y)}^{\sim})^{\alpha} W_{f(Y)},$$

s.t.

$$\begin{aligned} (W_{f(X)})^T f(X) f(X)^T W_{f(X)} &= I, \\ (W_{f(Y)})^T f(Y) f(Y)^T W_{f(Y)} &= I. \end{aligned} \tag{10}$$

where I is an identity matrix.

The total discriminant correlation between $f(X)$ and $f(Y)$ in DDFCCA is written in equation (11)

$$\text{corr}(f(X), f(Y)) = \text{tr}(T'T)^{1/2}, \tag{11}$$

where tr is the trace of a matrix, and

$$T = C_{f(X)f(X)}^{-\frac{1}{2}} \cdot (C_{f(X)f(Y)}^{\sim})^{\alpha} \cdot C_{f(Y)f(Y)}^{-\frac{1}{2}}, \tag{12}$$

DDFCCA (6)

where $C_{f(X)f(X)}$ and $C_{f(Y)f(Y)}$ are the within-correlation matrices of two variable sets $f(X)$ and $f(Y)$, and the singular value decomposition of T is given as $T = A \cdot E \cdot B'$.

The gradient of $\text{corr}(f(x), f(y))$ is calculated as below

$$\begin{aligned} & \frac{\partial \text{corr}(f(X), f(Y))}{\partial f(X)} \\ &= \frac{\partial \text{tr}(T' T)^{\frac{1}{2}}}{\partial f(X)} \\ &= \frac{1}{N-1} (2 \nabla_{f(X)f(X)} f(X) + \nabla_{f(X)f(Y)} f(Y)) \end{aligned} \quad (13)$$

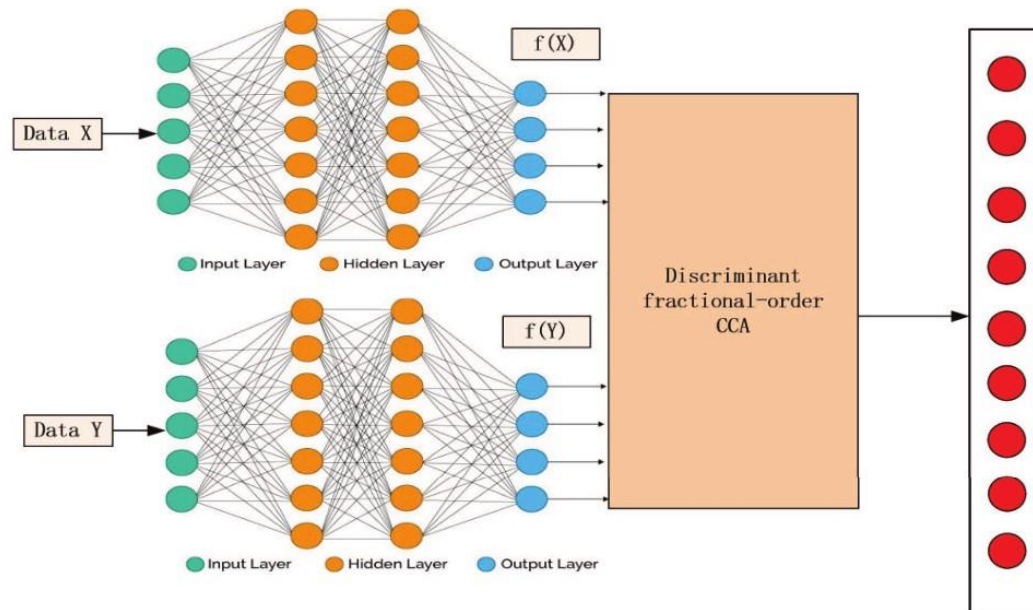
where

$$\begin{aligned} \nabla_{f(X)f(X)} &= -\frac{1}{2} C_{f(X)f(X)}^{-\frac{1}{2}} \cdot A \cdot E \cdot A' \cdot C_{f(X)f(X)}^{-\frac{1}{2}}, \\ \nabla_{f(X)f(Y)} &= C_{f(X)f(X)}^{-\frac{1}{2}} \cdot A \cdot B' \cdot C_{f(Y)f(Y)}^{-\frac{1}{2}}. \end{aligned} \quad (14)$$

DDFCCA (7)

- Deep denoising autoencoder [9] – initialize the values of parameters in the NNs.
- Limited memory-Broyden-Fletcher-Goldfarb-Shanno optimization function [10] – minimize the local reconstruction error with a quadratic penalty.

The representation of the proposed DDFCCA model is depicted in Figure. 1.



Experiments and Analysis(1)

The performance of DDFCCA is evaluated on three recognition tasks:

- handwritten digit recognition,
- audio emotion recognition,
- object recognition.

The value of α is set to ($\alpha = 0.1, 0.2, \dots, 1$) and the optimal results are reported.

Experiments and Analysis (2)

Handwritten digit recognition-the MNIST database

- 60000 training images and 10000 testing images.
- To avoid over-fitting, 10000 images from the training subset are used for tuning purpose.
- All samples are normalized to 28×28 pixels of binary values.
- The images are divided into 2 parts (views): right half and left half of 14 columns each.
- DDFCCA is performed on the two parts (views) and the optimal result is shown in TABLE I.
- The optimal result achieves with a 3-layer cascade network (1024-1024-50).

TABLE I
EXPERIMENTAL RESULTS ON THE HANDWRITTEN
DIGIT MNIST DATABASE

Method	Recognition Accuracy
KCCA [2]	86.51%
Photonics-enabled CNN [11]	90.04%
DW-ELM-AE [12]	96.62%
DCCA [3]	96.87%
DCFA [13]	97.21%
DDCCA [14]	97.24%
DCCF [15]	97.41%
DDFCCA	98.43%

Experiments and Analysis (3)

Audio emotion recognition-the RML emotional database

- Select 76 samples from each emotional state, resulting in 456 samples (76 per/emotion *6 emotions=456).
- For each emotion, 60 samples are for training and rest for testing.
- In total, 360 samples for training and 96 for testing
- Two audio features, Prosodic and mel frequency cepstral coefficient (MFCC), are extracted
- Individual recognition accuracies are given in TABLE II.

TABLE II
THE RECOGNITION ACCURACY OF A SINGLE FEATURE
ON AUDIO EMOTION RECOGNITION

Feature	Recognition Accuracy
Prosodic	51.04%
MFCC	37.50%

Experiments and Analysis (4)

- The two audio features fused by DDFCCA with the result shown in TABLE III.
- Optimal result obtained with a 3-layer cascade network (180-180-90).
- Performance by SOTA methods also tabulated for comparison.

TABLE III
THE RECOGNITION ACCURACY OF DIFFERENT METHODS ON AUDIO EMOTION RECOGNITION

Method	Recognition Accuracy
VGG-16 [16]	43.58%
KCCA [2]	57.29%
PNCC [17]	58.33%
DCCA [3]	59.46%
<i>Alexnet</i> [18]	59.46%
ALP Two-stage [19]	61.35%
DCNN-DTPM [20]	62.40%
DCA [21]	63.45%
DCFA [13]	63.54%
L-GrIN [22]	65.50%
Complete KECA+LDA [23]	65.63%
DDCCA [14]	65.63%
DDFCCA	68.75%

Experiments and Analysis (5)

Object recognition-the Caltech101 database

- 30 images from each class are chosen as training samples and the remaining images are for testing.
- Two fully connected layers fc6, and fc7 of a Alexnet architecture are employed for feature extraction.
- The classification accuracies of the two layers are given in TABLE IV.

TABLE IV
RECOGNITION ACCURACY OF A SINGLE DNN BASED
FEATURE (CALTECH 101)

Feature	Recognition Accuracy
fc6	77.84%
fc7	77.65%

Experiments and Analysis (6)

- The extracted features by the two layers are fed into DDFCCA for information fusion.
- The optimal result of DDFCCA achieves with a 3-layer cascade network (1000-1000-100) as shown in TABLE V.
- Performance by SOTA methods also tabulated for comparison.

TABLE V
RECOGNITION ACCURACY OF DIFFERENT METHODS
(CALTECH 101)

Methods	Recognition Accuracy
RNPCANet [24]	72.27%
SRAAL [25]	90.00%
PCANet [26]	89.98%
CFMNN [27]	89.36%
ResNet-101(self-tuning) [28]	89.30%
VGG16 [28]	87.30%
FSIL [29]	86.44%
DSDPL [30]	76.76%
SDADL [31]	76.28%
Superpixels-feature Fusion [32]	74.50%
AHG-JLDE [33]	80.10%
Spatial Pooling [34]	82.45%
MKL-SRC [35]	80.61%
Fcss [36]	83.00%
CovLets [37]	74.70%
FPNN [38]	88.20%
DDFCCA	90.21%

Summary

1. A discriminant fractional-order canonical correlation analysis (DDFCCA) method is proposed with application to information fusion.
2. Incorporated into a NN-based architecture, the fractional-order based discriminant power generates high quality representations of feature fusion.
3. Experimental results show the superiority of the proposed DDFCCA method.

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Thanks!